C2 JUNE 2012

1) $(2-3x)^5$

$= 2^5 + \binom{5}{1}2^4(-3x) + \binom{5}{2}2^3(-3x)^2$

$= 32 - 240x + 720x^2$

2a) $2\log_3 x - \log_3(x-2) = 2$

$\log_3 x^2 - \log_3(x-2) = 2$

$\log_3\left(\frac{x^2}{(x-2)}\right) = 2$

$\frac{x^2}{x-2} = 3^2 = 9$

$x^2 = 9(x-2)$

$x^2 - 9x + 18$

$(x-6)(x-3)$

$\underline{x=6}$ or $\underline{x=3}$

3 a) $x^2 + y^2 - 20x - 16y + 139 = 0$

$(x-10)^2 - 100 + (y-8)^2 - 64 + 139 = 0$

$(x-10)^2 + (y-8)^2 - 25 = 0$

$(x-10)^2 + (y-8)^2 = 5^2$

centre $= (10, 8)$

b)

$r^2 = 100 + 64 - 139 = 25$

$r = \sqrt{25} = \underline{5}$

c) $(x-10)^2 + (y-8)^2 = 25$

when $x = 13$

$(3)^2 + (y-8)^2 = 25$

$y^2 - 16y + 64 = 16$
$y^2 - 16y + 48 = 0$
$(y-4)(y-12)$
$y = 4$ or $y = 12$

d) arc length $= r\theta = 5 \times 1.855 = 9.275$

$9.275 + 2r = 9.275 + 10$
$= \underline{19.275}$

4 a) $f(x) = 2x^3 - 7x^2 - 10x + 24$

$f(-2) = 2(-2)^3 - 7(-2)^2 - 10(-2) + 24$
$= -16 - 28 + 20 + 24$
$= 0$
$\therefore$ $(x+2)$ must be a factor

b)

$$\begin{array}{r|l} & 2x^2 - 11x + 12 \\ \hline x+2 & 2x^3 - 7x^2 - 10x + 24 \\ & 2x^3 + 4x^2 \\ & \quad -11x^2 - 10x \\ & \quad -11x^2 - 22x \\ & \qquad 12x + 24 \\ & \qquad 12x + 24 \\ \hline & \qquad\qquad 0 \end{array}$$

$(x+2)(2x^2 - 11x + 12)$

$\boxed{(x+2)(x-4)(2x-3)}$

5a) $y = 10 - x$

$y = 10x - x^2 - 8$

$$10 - x = 10x - x^2 - 8$$
$$x^2 - 11x + 18 = 0$$
$$(x-9)(x-2) = 0$$
$$x = 9 \text{ or } x = 2$$

when $x = 9$

$y = 10 - 9$

$y = 1$

B $(9, 1)$

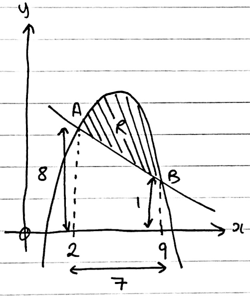
when $x = 2$

$y = 10 - 2$

$y = 8$

A $(2, 8)$

b)



Area R $= \int_2^9$ curve − trapezium

$$= \int_2^9 10x - x^2 - 8 \, dx$$
$$= \left[5x^2 - \frac{1}{3}x^3 - 8x\right]_2^9$$
$$= 90 - \frac{4}{3} = \frac{266}{3}$$

trapezium

$\frac{1}{2}(a+b)h$

$$= \frac{1}{2}(1+8)7 = \frac{63}{2}$$

$$R = \frac{266}{3} - \frac{63}{2} = \boxed{\frac{343}{6}}$$

6a) $\tan 2x = 5\sin 2x$

$$\frac{\sin 2x}{\cos 2x} = 5\sin 2x$$

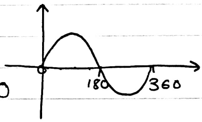
$$\sin 2x = 5\sin 2x\cos 2x$$

$$\sin 2x - 5\sin 2x\cos 2x = 0$$

$$(1-5\cos 2x)\sin 2x = 0$$

b) either $\sin 2x = 0$

or $(1-5\cos 2x) = 0$



$$\sin 2x = 0$$

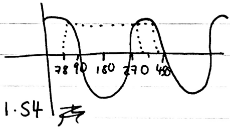
$$2x = \sin^{-1}(0) = 0, 180, 360$$

$$\underline{x = 0, 90, 180}$$

or

$$1-5\cos 2x = 0$$

$$\cos 2x = \frac{1}{5}$$



$$2x = 78.46, 281.54$$

$$\underline{x = 39.2, 140.8}$$

7a) $y = \sqrt{3^x + x}$

x	0	0.25	0.5	0.75	1
y	1	1.251	1.494	1.741	2

b)

$$\frac{1}{2}0.25\left[(1+2) + 2\times(1.251 + 1.494 + 1.741)\right]$$

$$= \frac{1}{8}\left[3 + 2\times 4.486\right]$$

$$= \underline{1.4965}$$

8a) $\pi x^2 h = 60$

$h = \frac{60}{\pi x^2}$

b) $2 \times \pi x^2 + 2\pi x \times h$

$= 2\pi x^2 + 2\pi x\left(\frac{60}{\pi x^2}\right)$

$A = 2\pi x^2 + \frac{120}{x}$

c) $\frac{dA}{dx} = 4\pi x - \frac{120}{x^2}$

at min = 0 $\quad 4\pi x = \frac{120}{x^2}$

$x^3 = \frac{120}{4\pi}$

$x = \sqrt[3]{\frac{120}{4\pi}} \approx 2.12$

d) min value of A

$= 2\pi(2.12)^2 + \frac{120}{(2.12)} = \underline{85}$

e) $\frac{d^2A}{dx^2} = 4\pi + \frac{240}{x^3} > 0$

$\therefore \frac{d^2A}{dx^2} > 0 \quad \therefore$ A is minimum

9a) $S_n = a + ar + ar^2 \cdots ar^{n-2} + ar^{n-1}$

$rS_n = ar + ar^2 + ar^3 \cdots ar^{n-1} + ar^n$

$S_n - rS_n = S_n(1-r) = a - ar^n = a(1-r^n)$

$$S_n = \frac{S_n(1-r)}{1-r} = \frac{a(1-r^n)}{1-r}$$

b) $ar^2 = 5.4$ $\qquad$ $ar^{\cancel{3}4} = 1.944$

$a = \frac{5.4}{r^2}$ $\qquad$ $a = \frac{1.944}{r^4}$

equate

$$\frac{5.4}{r^2} = \frac{1.944}{r^4}$$

$$5.4r^2 = 1.944$$

$$r^2 = \frac{9}{25}$$

$$\boxed{r = \frac{3}{5}}$$

c) $a = \dfrac{5.4}{\left(\frac{3}{5}\right)^2} = \underline{15}$

d) $S_\infty = \dfrac{a}{1-r} = \dfrac{15}{1-\frac{3}{5}} = \dfrac{75}{2} = \underline{37.5}$